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Charged black holes in generalized dilaton-axion gravity

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Abstract: We study generic Einstein-Maxwell-Kalb-Ramond-dilaton actions, and derive conditions under which they give rise to static, spherically symmetric black hole solutions. We obtain new asymptotically flat and non-flat black hole solutions which are in general electrically and magnetically charged. They have positive definite and finite quasi-local masses. Existing non-rotating black hole solutions (including those appearing in low energy string theory) are recovered in special limits.

Keywords: Black Holes, Black Holes in String Theory, Superstrings and Heterotic Strings.

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1. Introduction

Theories of gravity, with scalar and/or pseudoscalar field(s) in the background, have been under investigation for a long time. Such theories, taken in conjunction with the couplings of the (pseudo)scalar field(s) with vector or tensor gauge fields, turn out to be very promising to obtain various black hole solutions of the gravitational field equations. For gravity coupled minimally to one or more scalar field(s), it has long been established [1], as an immediate follow-up of the 'no hair' theorem, that a black hole cannot support the additional scalar degree(s) of freedom in a static spherically symmetric spacetime which is required to be asymptotically flat. This is true for all scalar fields, and evidently for all pseudoscalar fields as well, regardless of their mass or any positive (convex) potential assigned to them. Attempts have therefore been made either to explore non-minimal (conformal) scalar couplings with gravity [2], or to invoke a non-vanishing cosmological constant within the usual minimal coupling prescription [3], in order to obtain regular and stable black hole solutions of the effective Einstein's equations. It has further been shown that in presence of both gauge fields as well as scalar/pseudoscalar fields in a curved background, black hole solutions exist for some specific choices of the couplings between the (pseudo)scalar(s) and the gauge fields. Such couplings arise naturally in some string theories and are responsible for the existence of black holes in such theories. There have been extensive investigations for black holes in string theory (see [4] for some comprehensive reviews) and the implications of the typical (pseudo)scalar-gauge field couplings (which are intimately associated with the duality relationships) have been studied in detail both in asymptotically flat (AF) spacetimes [5]-[6] as well as in asymptotically non-flat (AN) spacetimes [7]-[8]. These black hole solutions, characterized by the gauge charges, can be useful in exploring the effects of string theory in the low energy world.

In this paper we examine the possibility of the existence of charged black holes in a theory of gravity coupled with scalar/pseudoscalar fields and a U(1) (electromagnetic) gauge field. Our approach is to start with arbitrary scalar and pseudoscalar couplings with the invariant quantities $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}\,{}^\star F^{\mu\nu}$, in the action (where $F_{\mu\nu}$ is the electromagnetic field strength and ${}^\star F^{\mu\nu}$ is its dual). From such a general action we try to construct charged black hole solutions of the gravitational field equations in four dimensions. Requiring the existence of a horizon, we determine the nature of the scalar-electromagnetic couplings. This in a way generalizes the well-known scalar-electromagnetic coupling in string theory leading to black hole solutions.

As is well known [9, 10], the massless spectrum of the bosonic sector of string theory consists of the graviton, the vector gauge fields, the second rank antisymmetric tensor Kalb-Ramond (KR) field [11] and the scalar dilaton. The dilaton has an exponential coupling with all the other fields in the action. The four dimensional effective action a theory of gravity, derived from string theory, is thus the standard Einstein-Hilbert action coupled to the dilaton ϕ, the KR field $B_{\mu\nu}$ and a set of U(1) gauge fields A^i_μ ($i = 1, \ldots, N$). For simplicity we consider only one U(1) gauge field A_μ — the Maxwell (electromagnetic) field. The KR field strength is, however, augmented with the U(1) Chern-Simons term in order to keep the corresponding quantum theory free from gauge anomaly.[1] The Chern-Simons term is found to play a crucial role in preserving gauge symmetry under both U(1) and KR gauge transformations [12]. Further, the Chern-Simons augmented third rank field strength corresponding to the second rank KR field can be expressed as a hodge dual of the derivative of a pseudoscalar field (called axion ξ) in four dimensions.

The gauge-invariant KR-electromagnetic coupling, which appears as a $\xi F\,{}^\star F$ term in the effective action, has an interesting astrophysical/cosmologicalimplication, namely cosmic optical birefringence. This has been studied in detail in some recent works both in four dimensions [13] and also in the context of higher dimensional Randall-Sundrum [14] scenario [15 − 17]. The effect of the KR field on various other physical phenomena, such as gravitational lensing and redshift [18], neutrino oscillation [19], Cosmic Microwave polarization anisotropy [20], cosmic acceleration [21] etc. have also been explored to look for possible indirect evidences of string theory at low energies. On the other hand, the effects of the dilaton are of substantial interest both in cosmology (see [22] for some reviews) as well in the context of charged black holes and their implications in string theory [4].

In this paper we focus on charged black hole solutions in a KR-dilaton background. We resort to a string inspired scenario in which the standard Einstein-Maxwell theory is supplemented by two scalar/pseudoscalar fields ϕ and ξ, which we later identify as the

[1] There is an additional augmentation of the KR field strength with the Lorentz Chern-Simons term on account of gravitational anomaly cancellation. However, we are not taking any notice of it since it is dimensionally suppressed, for being quadratic in the scalar curvature. In fact, we are not taking into account the higher curvature corrections to the tree level string action anyway.

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stringy dilaton and axion fields in four dimensions. We consider generic couplings (as arbitrary functions of ϕ and ξ) between these (pseudo)scalars and the electromagnetic field and also take into account an arbitrary interaction between ϕ and ξ. With this setup, in section 2, we arrive at an effective action that involves a single scalar field ψ (which is a function of ϕ and ξ) coupled to the Einstein-Maxwell system. Resorting to a generic ansatz for the metric, we obtain exact solutions of the gravitational field equations, representing static spherically symmetric charged black holes. In the process, the general forms of the effective scalar ψ and its coupling with the electromagnetic field is determined in terms of the radial variable r. It turns out that the asymptotic properties of solutions (AF or AN) depends upon some free parameters present in our ansatz. For both AF and AN types of spacetimes, we work out the general expressions for the gravitating mass m ($\equiv GM$) following the standard quasi-local mass formalisms [23, 24] in section 3. In section 4, we obtain the solution for the metric of both AF and AN types and that for the effective scalar field ψ. Finally, in section 5 we analyze our solutions from the point of view of the low energy effective string theory (as well as some possible generalizations of this) by varying the parameters of the action. The possible origins of these parameters are explained in section 5. We work out the complete set of solutions separately for both AF and AN spacetimes for some specific values of these parameters. In this process we reproduce some known solutions in dilaton-axion gravity as well as find some new ones.

It is also to be mentioned here that in most of this paper, we work in the Einstein frame, in which the scalars are minimally coupled to gravity and the Planck length, which is proportional to square root of the universal gravitational constant G, is taken as the fundamental length scale. In dealing with the string background geometries, it is in fact always convenient to work in the Einstein frame, rather than the string frame which is related to the Einstein frame via the conformal transformation[2] $g_{\mu\nu} = e^{-\phi}\tilde{g}_{\mu\nu}$, where $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are respectively the Einstein metric and the string metric. In the string frame, the string length scale l_s is taken as the fundamental unit for all measurements, while the effective Planck scale varies with the dilaton. As such, there is no usual way of a physical realization of any experimental data in such a frame.

2. General formalism

Let us begin with a string inspired scalar coupled Einstein-Hilbert-Maxwell action in four dimensions (in Einstein frame)

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\omega(\phi)}{2}\partial_\mu\xi\partial^\mu\xi - \alpha(\phi,\xi)F_{\mu\nu}F^{\mu\nu} - \beta(\phi,\xi)F_{\mu\nu}{}^\star F^{\mu\nu}\right]. \tag{2.1}$$

where $\kappa = 8\pi G$ is the four dimensional gravitational coupling constant. R is the curvature scalar, $F(= \partial_{[\mu}A_{\nu]})$ is the field strength of the Maxwell field A_ν, and $\{\phi,\xi\}$ are two massless scalar/pseudoscalar fields where ξ acquires a non-minimal kinetic term given by the function

[2] In our convention ϕ is the dilaton, rather than $\Phi = \phi/2$ as mostly in the literature.

$\omega(\phi)$ because of it's interaction with the scalar ϕ. In addition, the functions α and β describe the couplings of ϕ and ξ with the Maxwell field which at this stage are chosen to be as general as possible. The free Maxwell action consists of both the scalar and pseudoscalar invariants, $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}{}^{\star}F^{\mu\nu}$, where ${}^{\star}F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the (Hodge-) dual Maxwell field strength.

The field equations corresponding to the above action (2.1) are given by

$$
\begin{aligned}
R_{\mu\nu} &= \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{\omega(\phi)}{2}\partial_\mu\xi\partial_\nu\xi + \\
&\quad +2\alpha(\phi,\xi)\left(F_{\mu\rho}F^{\rho}_{\nu} - \frac{1}{4}g_{\mu\nu}F_{\rho\lambda}F^{\rho\lambda}\right) && (2.2)\\
\nabla_\mu\partial^\mu\phi &= \frac{1}{2}\partial_\phi\omega\partial_\mu\xi\partial^\mu\xi + \partial_\phi\alpha F_{\mu\nu}F^{\mu\nu} + \partial_\phi\beta F_{\mu\nu}{}^{\star}F^{\mu\nu} && (2.3)\\
\nabla_\mu\left[\omega(\phi)\partial^\mu\xi\right] &= \partial_\xi\alpha F_{\mu\nu}F^{\mu\nu} + \partial_\xi\beta F_{\mu\nu}{}^{\star}F^{\mu\nu} && (2.4)\\
\nabla_\mu\left[\alpha(\phi,\xi)F^{\mu\nu} + \beta(\phi,\xi)^{\star}F^{\mu\nu}\right] &= 0 && (2.5)\\
\nabla^{\star}_\mu F^{\mu\nu} &= 0\,. && (2.6)
\end{aligned}
$$

The last equation in this set is the Maxwell-Bianchi identity.

We study the regular non-rotating charged black hole solutions of these field equations subject to different limiting conditions. Our aim is to pinpoint the black hole families corresponding to typical forms of the couplings ω, α and β in terms of the scalars ϕ and ξ. In the specific case of low energy effective string theory in four dimensions, ϕ can be identified with the massless scalar dilaton while ξ to the pseudoscalar axion. The low energy effective string action in the Einstein frame corresponds to the above action (2.1) with $\omega = e^{2\phi}, \alpha = e^{-\phi}$ and $\beta = \xi$.

We look for static spherical symmetric black hole solutions with the line element of the form

$$
ds^2 = -N^2(r)dt^2 + \frac{dr^2}{N^2(r)} + R^2(r)d\Omega^2\,. \tag{2.7}
$$

where $d\Omega^2 = (d\vartheta^2 + \sin^2\vartheta d\varphi^2)$ is the metric on the two sphere. The scalar fields ϕ and ξ are functions of the radial variable r only, while the Maxwell's equations (2.2d) and (2.2e) can be integrated to yield the non-vanishing components

$$
\begin{aligned}
F_{\mu\nu} : F_{\mathrm{tr}} &= -F_{\mathrm{rt}} = \frac{q_e - q_m\beta}{\alpha R^2}\,dt\wedge dr \\
F_{\vartheta\varphi} &= -F_{\varphi\vartheta} = q_m\sin\vartheta\;d\vartheta\wedge d\varphi\,. && (2.8)
\end{aligned}
$$

The integration constants q_e and q_m can respectively be interpreted as the scalar field shielded electric and magnetic charges (as of the isolated charges corresponding to a system with a scalar "fluid") defined in the usual way (see [25]):

$$
q_e = -\frac{1}{4\pi}\int_S(\alpha^{\star}F + \beta F)\,; \qquad q_m = -\frac{1}{4\pi}\int_S F \tag{2.9}
$$

where the integrals are over a two-dimensional surface S, bounding a three-dimensional hypersurface Σ, pushed to spatial infinity. A combination of q_e and q_m may, in fact, be identified with the total (bare) electric and magnetic charges of the matter distribution once the asymptotic behaviour of the metric functions $R(r), N(r)$ and that of the couplings $\alpha(r), \beta(r)$ are ascertained. In asymptotically flat spacetimes, as will be shown later, such identifications are straightforward. However, in asymptotically non-flat spacetimes a complete knowledge of the solutions to the field equations is required in order to identify q_e and q_m as combinations of the (bare) electric and magnetic charges. We shall discuss this in detail in the following sections.

In the static spherical symmetric spacetime the field equations (2.2) reduce to

$$(R^2N^2)'' = 2 \tag{2.10}$$

$$\left(\frac{R'}{R}\right)' + \left(\frac{R'}{R}\right)^2 + \frac{1}{4}\left(\phi'^2 + \omega\xi'^2\right) = 0 \tag{2.11}$$

$$(R^2NN')' = \frac{1}{R^2}\left[q_m^2\alpha + \frac{(q_e - q_m\beta)^2}{\alpha}\right] \tag{2.12}$$

$$(R^2N^2\omega\xi')' = \frac{2}{R^2}\partial_\xi\left[q_m^2\alpha + \frac{(q_e - q_m\beta)^2}{\alpha}\right] \tag{2.13}$$

$$\begin{aligned}(R^2N^2\phi')' = &\frac{2}{R^2}\,\partial_\phi\left[q_m^2\alpha + \frac{(q_e - q_m\beta)^2}{\alpha}\right] + \\ &+\frac{1}{2}R^2N^2\xi'^2\partial_\phi\omega\end{aligned} \tag{2.14}$$

where a prime denotes derivative with respect to r. Eq. (2.10a) can at once be integrated to find that the product $R^2(r)N^2(r)$ is purely quadratic in r such that,

$$R^2(r)N^2(r) = (r - r_+)(r - r_-) \tag{2.15}$$

where $r_\pm$ are the constants of integration. Eq. (2.10b), on the other hand, is nothing but the trace equation corresponding to the Einstein's equations (2.2a), which is essentially the scalar curvature equation. One may verify explicitly that the possibility of obtaining a black hole solution of the field equations in a closed form depends entirely on the integrability of this equation (2.10b).

Now, it is easy to check that entire set of field equations (2.10) can be derived exactly from an effective action for Einstein-Maxwell theory coupled to a single scalar field ψ (which is defined in terms of ϕ and ξ as $\psi'^2 = \phi'^2 + \omega\xi'^2$):

$$S_{\text{eff}} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \alpha(\psi)F_{\mu\nu}F^{\mu\nu} - \beta(\psi)F_{\mu\nu}{}^\star F^{\mu\nu}\right] \tag{2.16}$$

whence the effective field equations are given by

$$(R^2N^2)'' = 2 \tag{2.17}$$

$$\left(\frac{R'}{R}\right)' + \left(\frac{R'}{R}\right)^2 + \frac{1}{4}\psi'^2 = 0 \tag{2.18}$$

$$(R^2 N N')' = \frac{1}{4R^2}\,\gamma \tag{2.19}$$

$$\left(R^2 N^2 \psi'\right)' = \frac{1}{2R^2}\,\partial_\psi \gamma \tag{2.20}$$

where

$$\gamma = 4\left[q_m^2 \alpha + \frac{(q_e - q_m \beta)^2}{\alpha}\right] \tag{2.21}$$

$$\psi'^2 = \phi'^2 + \omega(\phi)\xi'^2 \tag{2.22}$$

$$\partial_\xi \gamma = 2R^2 (R^2 N^2 \omega \xi')' \,. \tag{2.23}$$

The equation (2.17b) here is the scalar curvature equation which corresponds to the trace of the Einstein's equations. Charge neutrality of ψ suggests the following ansatz for the metric

$$R(r) = r\left(k + \frac{2r_0}{r}\right)^n \tag{2.24}$$

where k and r_0 are constant real parameters; k is dimensionless while r_0 has the dimension of r. From eq. (2.15) one therefore has

$$N(r) = \frac{\sqrt{(r - r_+)\,(r - r_-)}}{r\,(k + 2r_0/r)^n} \,. \tag{2.25}$$

It can be shown that eq. (2.17b) integrates to the following real solution for $\psi(r)$, provided n lies between 0 and 1:

$$\psi(r) = \psi_0 - 2\sqrt{n(1-n)}\;\ln\left(k + \frac{2r_0}{r}\right); \qquad (0 \le n \le 1) \tag{2.26}$$

where ψ_0 is a constant of integration. For the extreme cases $n = 0$ or 1, the field ψ is trivial and by suitably shifting the radial variable r by a constant parameter given in terms of k and r_0 one gets back the usual Reissner-Nordström (RN) solution of Einstein-Maxwell theory. In order to construct a solution with a non-trivial ψ, we therefore exclude these values and assume $0 < n < 1$.

The equations (2.17c) and (2.17d) together yield the differential equation for the effective coupling parameter γ

$$\partial_\psi^2 \gamma - \frac{1 - 2n}{\sqrt{n(1-n)}} \partial_\psi \gamma - \gamma = 0 \tag{2.27}$$

with solution

$$\gamma(r) = k_1 \exp\left[\sqrt{\frac{1-n}{n}}\psi(r)\right] + k_2 \exp\left[-\sqrt{\frac{n}{1-n}}\psi(r)\right] \tag{2.28}$$

where k_1 and k_2 are arbitrary constants. Using the above equation (2.26) for $\psi(r)$, we have

$$\gamma(r) = K_1 \left(k + \frac{2r_0}{r}\right)^{2(n-1)} + K_2 \left(k + \frac{2r_0}{r}\right)^{2n} \tag{2.29}$$

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where $K_1 = k_1 e^{\sqrt{(1-n)/n}\ \psi_0}$ and $K_2 = k_2 e^{-\sqrt{n/(1-n)}\ \psi_0}$. Substituting this expression for γ back in eqs. (2.17c) and (2.17d) and solving for K_1 and K_2, we obtain

$$K_1 = 4n\left[4r_0^2 + 2kr_0\left(r_+ + r_-\right) + k^2 r_+ r_-\right]; \qquad K_2 = 4\left(1-n\right) r_+ r_- . \tag{2.30}$$

Now, one can verify that $k \neq 0$ ($k = 0$) in eq. (2.24) corresponds to asymptotically flat (non-flat) spacetimes. For the AF case, one can always set $k = 1$, without loss of generality, to get black holes with spherical topology. In what follows, we determine the masses for both AF and AN solutions corresponding to the cases $k = 1$ and $k = 0$ respectively.

3. The mass formulae

We resort to the general formalism of quasi-local mass developed independently by Brown and York (BY) [23] and by Hawking and Horowitz (HH) [24], and further described in [7, 26, 27] in the context of charged black holes with curved asymptotes in (2+1) dimensional [26] and (3+1) dimensional dilaton gravity [7]. Conformal invariance of the quasi-local mass, according to both BY and HH prescriptions, has been studied in detail in [27] and alternative proposals have been put forward in [28]. General features of quasi-local mass of non-asymptotically flat dilaton black holes have also been discussed recently in [8]. We briefly outline the following rule:

In D dimensions, given a generic action functional of the metric $g^D_{\mu\nu}$ and scalar and other matter fields $\{\psi, F\}$:

$$S(g_D, \psi, F) = \frac{1}{2\kappa_D}\int_{\mathcal{M}} d^D x\sqrt{-g_D}\,U(\psi)\left[\mathcal{R}(g_D) - W(\psi)\partial_\mu\psi\partial^\mu\psi - V(\psi) + X(\psi)L_m(F)\right] \tag{3.1}$$

and a line element of the form

$$ds^2 = -N^2(R)dt^2 + \frac{dR^2}{Y^2(R)} + R^2 d\Omega^2_{D-2} \tag{3.2}$$

the quasi-local mass is defined as

$$M(R_b) = \frac{N(R_b)}{\kappa_D}\left[Y_0(R_b) - Y(R_b)\right]\left[\frac{d}{dR}\left\{A_{D-2}(R)U(\psi)\right\}\right]_{R=R_b} \tag{3.3}$$

where $\kappa_D = 8\pi G_D$ is the D dimensional gravitational coupling constant; $Y_0(R_b)$ is an arbitrary function which determines the zero of energy of a background spacetime; and R_b is the radius of the $(D-2)$ dimensional spacelike hypersurface boundary whose area is A_{D-2} — for a boundary with the topology of a sphere

$$A_{D-2} = (4\pi)^{\frac{D-2}{2}}\,\frac{\Gamma\left(\frac{D-2}{2}\right)}{\Gamma\left(D-2\right)}\,R_b^{D-2} = \int_{\mathcal{B}} d^{D-2}x\sqrt{\sigma} \tag{3.4}$$

where σ is the determinant of the metric on the boundary $\mathcal{B}$.

In four dimensions ($D = 4$), the area of a spherical hypersurface boundary is simply given by $A_{D-2} \equiv A_2 = 4\pi R_b^2$, and the expression (3.3) for the quasi-local mass reduces to

$$m(R_b) \equiv GM(R_b) = N(R_b)\left[Y_0(R_b) - Y(R_b)\right]\left[\frac{R^2}{2}\left(\frac{dU(\psi)}{dR} + \frac{2U(\psi)}{R}\right)\right]_{R=R_b}. \tag{3.5}$$

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Now, comparing the general action (3.1) with the effective action (2.16) for $D = 4$, we have $U(\psi) = 1$ and $W(\psi) = 1/2$, whence the quasi-local mass is given by

$$m(R_b) = R_b N(R_b) \left[Y_0(R_b) - Y(R_b) \right] . \tag{3.6}$$

Also, casting the metric (3.2) in the form (2.7), one has $Y = NdR/dr$. Setting $r = r_b$ corresponding to $R = R_b$, we have

$$m(r_b) = R(r_b) N(r_b) \left[Y_0(r_b) - Y(r_b) \right] . \tag{3.7}$$

If R is a monotonically increasing function of r, and vice versa, the actual gravitating rest mass $m (\equiv GM)$ may be obtained by taking the value of $m(r_b)$ in the limit $r_b \to \infty$, i.e., by pushing the hypersurface boundary to spatial infinity. Indeed in the present scenario, the function R, given by the ansatz (2.24), increases monotonically with r for all values of the parameters k and r_0, since as mentioned above the index n can only take values strictly between 0 and 1 for a real solution of the effective field ψ. Using eq. (2.25) for the metric function $N(r)$, we get

$$Y(r) = N(r) \frac{d}{dr} R(r) = \left(1 - \frac{2nr_0}{kr + 2r_0} \right) \sqrt{\left(1 - \frac{r_+}{r} \right) \left(1 - \frac{r_-}{r} \right)} . \tag{3.8}$$

As such, the expression for the function $Y_0(r_b)$, and hence the quasi-local mass $m(r_b)$, can be obtained in the AF ($k = 1$) and AN ($k = 0$) cases as follows:

The general expression for the background (or reference) line-element is given by

$$ds_0^2 = -N_0^2 dt^2 + \frac{dr^2}{Y_0^2} + r^2 d\Omega^2 . \tag{3.9}$$

(i) For the AF case ($k = 1$) the background may universally be chosen [27, 28] to be flat with a constant lapse

$$N_0 \equiv N(r_b) = \frac{\sqrt{(r_b - r_+)(r_b - r_-)}}{r_b (1 + 2r_0/r_b)^n} \tag{3.10}$$

as dictated by the HH prescription [24]; and $Y_0 = 1$. This background metric is in fact the solution of the field equations corresponding to the action (2.16) for suitable reference values of the scalar ψ and the gauge field F, viz., $\psi = \psi_0$ and $F = 0$ respectively. The quasi-local mass can be evaluated using eqs. (3.7), (3.8) and (2.25) as

$$m(r_b) = r_b \left[\sqrt{\left(1 - \frac{r_+}{r_b} \right) \left(1 - \frac{r_-}{r_b} \right)} - \left(1 - \frac{2nr_0}{r_b + 2r_0} \right) \left(1 - \frac{r_+}{r_b} \right) \left(1 - \frac{r_-}{r_b} \right) \right] . \tag{3.11}$$

Taking the limit $r_b \to \infty$, we get

$$m = \lim_{r_b \to \infty} m(r_b) = \frac{r_+ + r_-}{2} + 2nr_0 \tag{3.12}$$

which is the same as the gravitating mass one could have obtained in the Arnowitt-Deser-Misner (ADM) formalism in asymptotically flat spacetime.

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(ii) For the AN case ($k = 0$) there is no unique choice of the reference geometry. However, a simple choice [27, 28, 7] for the background spacetime is the line element obtained by setting the constants $r_\pm$ to zero in the (non-flat) space part of the original line element (2.7). In other words, the background metric is chosen to be that given by eq. (3.9) with

$$N_0 \equiv N(r_b) = (2r_0)^{-n} r_b^{n-1} \sqrt{(r_b - r_+)(r_b - r_-)} \qquad \text{and}$$
$$Y_0 \equiv Y(r_b)|_{r_\pm=0} = 1 - n \qquad \text{[from eq. (3.8)]} . \quad (3.13)$$

The quasi-local mass turns out to be

$$m(r_b) = (1-n) r_b \left[\sqrt{\left(1 - \frac{r_+}{r_b}\right)\left(1 - \frac{r_-}{r_b}\right)} - \left(1 - \frac{r_+}{r_b}\right)\left(1 - \frac{r_-}{r_b}\right) \right] \quad (3.14)$$

whence taking the limit $r_b \to \infty$ one obtains the gravitating rest mass

$$m = \lim_{r_b \to \infty} m(r_b) = \left(\frac{1-n}{2}\right)(r_+ + r_-) . \quad (3.15)$$

4. General structures of static spherically symmetric black hole solutions with or without flat asymptotes

In this section we outline the general forms of both AF and AN solutions representing static charged non-rotating black holes in scalar coupled Einstein-Maxwell theory in four dimensions.

4.1 Asymptotically flat solutions

Let us recall the whole set [(2.17) – (2.30)] of effective field equations and the solutions that have been obtained with the metric ansatz (2.24) in section 2. We set the parameter $k = 1$ and refer to the mass formula (3.12) given in section 3 for AF spacetimes. Solving the equations (2.30) one obtains the expressions for $r_\pm$:

$$r_\pm = -r_0 + \frac{1}{16r_0}\left(\frac{K_1}{n} - \frac{K_2}{1-n}\right) \pm \sqrt{r_0^2 + \frac{1}{256r_0^2}\left(\frac{K_1}{n} - \frac{K_2}{1-n}\right)^2 - \frac{1}{8}\left(\frac{K_1}{n} + \frac{K_2}{1-n}\right)} \quad (4.1)$$

whence the mass expression (3.12) reduces to

$$m = \frac{1}{16r_0}\left(\frac{K_1}{n} - \frac{K_2}{1-n}\right) + (2n-1)\, r_0 . \quad (4.2)$$

Under a coordinate shift $r + r_0 \to r$ and a redefinition $r_\pm + r_0 \Rightarrow r_\pm$, the line element retains its original form (2.7), however, with metric functions $N(r)$ and $R(r)$ given as

$$N(r) = \frac{\sqrt{(r - r_+)(r - r_-)}}{R(r)} , \qquad R(r) = (r - r_0)^{1-n}(r + r_0)^n \quad (4.3)$$

where

$$r_{\pm} = m_0 \pm \sqrt{m_0^2 + r_0^2 - \frac{1}{8}\left(\frac{K_1}{n} + \frac{K_2}{1-n}\right)} \tag{4.4}$$

with

$$r_0 = \frac{1}{16 m_0}\left(\frac{K_1}{n} - \frac{K_2}{1-n}\right); \qquad m_0 = m - (2n-1) r_0 \,. \tag{4.5}$$

Asymptotic expansion of (4.3), using (4.4) and (4.5) suggests that m is the ADM mass of the charged matter distribution. The solutions (2.26) for the effective scalar ψ and (2.29) for the effective coupling γ reduce respectively to

$$\psi(r) = \psi_0 + 2\sqrt{n\,(1-n)} \ln\left(\frac{r - r_0}{r + r_0}\right) \tag{4.6}$$

$$\gamma(r) = K_1 \left(\frac{r - r_0}{r + r_0}\right)^{2(1-n)} + K_2 \left(\frac{r - r_0}{r + r_0}\right)^{-2n} . \tag{4.7}$$

The Ricci, squared-Ricci and Kretschmann scalar invariants all fall off to zero as $r \to \infty$:

$$\mathcal{R} \sim \frac{1}{r^4}\,; \qquad \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} \sim \frac{1}{r^8}; \qquad \mathcal{K} \equiv \mathcal{R}_{\mu\nu\lambda\sigma}\mathcal{R}^{\mu\nu\lambda\sigma} \sim \frac{1}{r^6}\,.$$

In fact for large r, the static spherical symmetric metric given in eq. (4.3) can always be cast in the form $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)$ along either spatial or null directions, $\eta_{\mu\nu}$ being the minkowskian. Also, equations (4.6) show that the effective scalar ψ as well as the effective coupling γ, tend toward constant values (ψ_0 and $K_1 + K_2$ respectively) as $r \to \infty$, thus verifying that the spacetime is asymptotically flat.

Now, the conditions for existence of regular black hole event horizon(s) can be given as follows:

- Firstly, the positivity of the mass m imposes a restriction the value of the parameter m_0:

$$m_0 > (1 - 2m)\, r_0 \,. \tag{4.8}$$

- Secondly, the term under the square root in the expression (4.4) must be non-negative. Using eqs. (4.5), this implies

$$(m_0 - r_0)^2 \geq \frac{K_2}{4\,(1-n)}. \tag{4.9}$$

- Finally, in order that at least one horizon exists, one must have $r_+ > r_0$, otherwise the metric function $N(r)$, given by eq. (4.3), will blow up before a radially infalling particle reaches the surface $r = r_+$ from infinity. Indeed, $r = r_0$ is a curvature singularity since all the scalar invariants $\mathcal{R}, \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}$ and $\mathcal{R}_{\mu\nu\alpha\beta}\mathcal{R}^{\mu\nu\alpha\beta}$ diverge therein, whereas the metric function $R(r = r_0)$ vanishes, as $0 < n < 1$ [See eqs. (4.3)]. Now, $R(r)$ is the radius of the two-spherical hypersurface of the spacetime given by the line element (2.7). Therefore, as R vanishes at $r = r_0$, the two-sphere crushes to a point,

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thereby implying that the singularity at $r = r_0$ is a 'point' (curvature) singularity. Such a singularity must obviously be naked unless at least there is a horizon at $r_+ (> r_0)$. From eqs. (4.4) and (4.5) this means

$$(m_0 - r_0) + \sqrt{(m_0 - r_0)^2 - \frac{K_2}{4(1-n)}} > 0\,. \tag{4.10}$$

Let us consider separately the following situations:

(i) $m_0 > r_0$: The condition (4.8) is satisfied automatically for any n greater than zero, which is always true of course. The condition (4.9) implies

$$m_0 \geq r_0 \pm \frac{1}{2}\sqrt{\frac{|K_2|}{1-n}}\,.$$

The inequality for the lower $(-)$ sign is always valid since we are considering the case $m_0 > r_0$. However, the upper $(+)$ sign sets a lower bound (LB) on m_0:

$$m_0^{LB} = r_0 + \frac{1}{2}\sqrt{\frac{|K_2|}{1-n}} \tag{4.11}$$

which, in turn, implies a lower bound on the mass m:

$$m^{LB} = 2nr_0 + \frac{1}{2}\sqrt{\frac{|K_2|}{1-n}}\,. \tag{4.12}$$

In any case, regardless of this bound, the condition (4.10) is always fulfilled whenever $m_0 > r_0$. However, the above bound on the mass definitely ensures that a blackhole horizon exists for $m \geq m^{LB}$.

(ii) $m_0 = r_0$: The condition (4.8) implies $n > 0$, which is always valid. However, since $n < 1$, both the conditions (4.9) and (4.10) are satisfied, and as such an event horizon exists, only for $K_2 < 0$.

(iii) $m_0 < r_0$: The condition (4.8) again implies $n > 0$, which is true. The condition (4.9) is satisfied for

$$r_0 \geq m_0 \pm \frac{1}{2}\sqrt{\frac{|K_2|}{1-n}}\,.$$

The inequality for the lower $(-)$ sign is always valid for any m_0 less than r_0; however, the upper $(+)$ sign sets an upper bound (UB) on m_0:

$$m_0^{UB} = r_0 - \frac{1}{2}\sqrt{\frac{|K_2|}{1-n}} \tag{4.13}$$

that is, an upper bound on the mass m:

$$m^{UB} = 2nr_0 - \frac{1}{2}\sqrt{\frac{|K_2|}{1-n}}\,. \tag{4.14}$$

The condition (4.10) further imposes the restriction $K_2 < 0$. Thus on the whole, the existence of a blackhole horizon for this case is ensured definitely when $m \leq m^{UB}$ and $K_2 < 0$.

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In section 5, in order to find the complete solution, we find expressions (as functions of r) for our original set of scalar fields $\{\phi, \xi\}$, from the set of solutions (4.3) – (4.6). We also relate the constants K_1, K_2 with the electromagnetic charges q_e, q_m and the asymptotic values (ϕ_0 and ξ_0, say) of the scalars ϕ and ξ. To this end, we check the compatibility of the entire set (4.3) – (4.6) with the set of equations (2.21) and solve the latter for ϕ and ξ.

However, the issue is easily settled in a specific situation where the functional forms of the couplings α and β (and as such γ) are known in terms of ϕ and ξ. We discuss this in detail in section 5 as we analyze the solutions for various cases corresponding to the specific scenario of dilaton-axion coupled Einstein-Maxwell theory.

4.2 Asymptotically non-flat solutions

Let us again refer to the set [(2.17) – (2.30)] of effective field equations and solutions obtained in section 2. We consider the case $k = 0$ in the solution ansatz (2.24), and resort to the corresponding expression (3.15) for the gravitating mass derived in section 3. The metric functions $N(r)$ and $R(r)$ are given by

$$N(r) = \frac{\sqrt{(r - r_+)(r - r_-)}}{R(r)}, \qquad R(r) = r \left(\frac{2r_0}{r}\right)^n. \tag{4.15}$$

Solving simultaneously the equations (2.30) (for $k = 0$) and (3.15), we obtain

$$r_\pm = \left(\frac{1}{1-n}\right) \left[m \pm \sqrt{m^2 - (1-n)\frac{K_2}{4}}\right]. \tag{4.16}$$

Since the parameter n lies between 0 and 1, one can check explicitly the large r fall-offs of all the scalar invariants

$$\mathcal{R} \sim r^{2(n-1)}; \qquad \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} \sim r^{4(n-1)}; \qquad \mathcal{K} \equiv \mathcal{R}_{\mu\nu\lambda\sigma}\mathcal{R}^{\mu\nu\lambda\sigma} \sim r^{4(n-1)}.$$

However, the spacetime is not asymptotically flat since the metric component $|g_{tt}| = |g^{rr}| = N^2(r)$ goes as r^{2n} in the limit $r \to \infty$. The solutions (2.26), for the effective scalar ψ, and (2.29), for the effective coupling γ, which now reduce to

$$\psi(r) = \psi_0 - 2\sqrt{n(1-n)}\, \ln\left(\frac{2r_0}{r}\right) \tag{4.17}$$

$$\gamma(r) = \left(4nr^2 + K_2\right) \left(\frac{2r_0}{r}\right)^{2n}, \tag{4.18}$$

also diverge as $r \to \infty$. Note that in deriving eq. (4.17b) we have substituted $16nr_0^2$ for K_1, following eq. (2.30).

The possible lower bound on the mass m for the existence of event horizon(s) can be obtained from eqs. (4.16):

$$m^{LB} = \frac{1}{2}\sqrt{(1-n)\,|K_2|}. \tag{4.19}$$

Moreover, the divergent behaviour of the effective scalar ψ and the effective coupling γ as $r \to \infty$, is reflected in similar divergences of the original set of scalar fields $\{\phi, \xi\}$ in the asympototic limit, which we shall show later on in the context of dilaton-axion blackholes.

In order to determine the precise functional forms of the scalar fields $\{\phi, \xi\}$ and to ascertain the value of the arbitrary constant K_2, we again need to check the compatibility of the entire set of solutions (4.15) – (4.17) with the equations (2.21) and solve the latter in some specific circumstances where the functional forms of the couplings α and β (and as such γ) are known in terms of ϕ and ξ. In the following section we resort to the dilaton-axion coupled Einstein-Maxwell theory, from the perspective of low energy effective string theory (with or without certain parametric modifications/generalizations), and study the charged black hole solutions corresponding to some typical special cases.

5. Charged black holes in Einstein-Maxwell-dilaton-axion theory

Let us refer to a generalized action for Einstein-Maxwell theory in four dimensions, coupled to the massless scalar dilaton ϕ and the massless pseudoscalar axion ξ in Einstein frame:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}e^{2a\phi}\partial_\mu\xi\partial^\mu\xi\right) - e^{-a\phi}F_{\mu\nu}F^{\mu\nu} - b\xi F_{\mu\nu}\,{}^\star F^{\mu\nu}\right]. \tag{5.1}$$

where a and b are two constant free parameters, the presence of which generalizes the low energy effective string action compactified to four spacetime dimensions. Both the dilaton and the axion fields are kept dimensionless.

For the ten dimensional low energy heterotic superstring compactified on a six-torus, the bosonic part of the effective action corresponds to (5.1) with the parameters $a = b = 1$. The axion ξ is connected via a duality in four dimensions with the three form $H_{\mu\nu\lambda}$, which is the field strength corresponding to the Kalb-Ramond (KR) two form $B_{\mu\nu}$, appearing in the string massless spectrum, augmented with the Chern-Simons term $A_{[\mu}F_{\nu\lambda]}$ that is required to keep the underlying theory free from gauge anomaly:

$$H_{\mu\nu\lambda} = \partial_{[\mu}B_{\nu\lambda]} + 2\sqrt{2\kappa}A_{[\mu}F_{\nu\lambda]} = \frac{e^{2a\phi}}{2\kappa}\,\epsilon^\sigma_{\mu\nu\lambda}\partial_\sigma\xi\,. \tag{5.2}$$

The four dimensional heterotic string action is expressed in terms of the dilaton ϕ and the three form $H_{\mu\nu\lambda}$ as

$$S_H = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) - \frac{1}{12}e^{-2\phi}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - e^{-\phi}F_{\mu\nu}F^{\mu\nu}\right] \tag{5.3}$$

the effective version of which in terms of ϕ and ξ is that given in eq. (5.1) with the parameters a and b set to unity. In our subsequent analysis, we shall be referring both to this conventional string theoretic picture, and the generalized scenarios where either a or b or both differ(s) from unity.

Deviations from usual the four dimensional low energy effective string action (5.3) primarily arise through the typical schemes of compactification (inspite of the standard Kaluza-Klein dimensional reduction) of the ten dimensional superstring to four spacetime dimensions. The invariance of the string effective equations of motion under certain symmetry groups and the duality conjectures, viz., both T-duality and strong-weak coupling duality, have been exploited in the context of different compactification schemes in order to develop the generating technique of non-trivial solutions from known field configurations [29]-[30]. In fact, the dual rotations and constant shifts in the axion that generates the $SL(2,\mathbb{R})$ group of symmetries of the equations of motion of dimensionally reduced heterotic string in the perturbative sector, have been shown to inflict a mutual inter-rotation of electric and magnetic charges as well as the dilaton and axion charges. This, in turn, enables one to obtain new solutions with non-trivial axion and dilaton starting from known ones with only non-trivial dilaton field but no axion [29, 31, 32]. With the involvement of scalar and vector moduli fields, as shown in [31, 30], there may still remain a symmetry of the equations of motion which can be utilized in the solution generating process. In a plethora of works [31]-[33, 34]-[35] such new solutions have been explored which in occasions describe regular black holes and/or black strings with various characterizations — charged, rotating or non-rotating — in presence of the dilaton and may also the axion (or, else the KR field) in different spacetime dimensions. We, however, in this paper confine our study to some parametric generalizations of the usual string picture, described by the action (5.3), through the introduction of the parameters a and b. The motivations behind such generalizations are as follows:

The presence of the parameter a, which we assume to be a real and non-negative constant, may be considered as a regulator for the strength of dilaton coupling with the Maxwell field. Such a parameter was originally taken into account in the works of Gibbons and Maeda [5, 36] in the context of dilaton black holes and membranes in higher dimensions. A number of interesting special cases may be pointed out:

- The case $a = 1$, as mentioned above, corresponds to the field theoretic limit of the ten dimensional or the effective four dimensional superstring model, in the bosonic sector.
- The case $a = \sqrt{1 + 2/n}$ corresponds to the four dimensional Kaluza-Klien toroidal reduction of a $4 + n$ dimensional theory.
- The case $a = 0$, corresponds to the usual Einstein-Maxwell theory coupled minimally with a massless Klien-Gordon scalar field ϕ, whenever the presence of the other scalar ξ (or, else the KR three form $H_{\mu\nu\lambda}$) is ignored.

The appearance of b may be related to the fact that the KR three tensor $H_{\mu\nu\lambda}$ can be interpreted as the torsion of background spacetime. Following the formalism in [12] one can identify $H_{\mu\nu\lambda}$ with a completely antisymmetric torsion tensor $T_{\mu\nu\lambda}$, modulo some multiplicative constant which regulates the strength of torsion-KR coupling. The presence of the Chern-Simons term in the definition of $H_{\mu\nu\lambda}$ is crucial in restoring the U(1) gauge invariance, which is apparently lost in the coupling of torsion with the U(1) gauge field [12]. Symbolically, $T_{\mu\nu\lambda}$ = constant $\times H_{\mu\nu\lambda}$, and the effective four dimensional version of the theory in terms of the dual axion field ξ, can in general involve a constant parameter in front of the kinetic term of ξ as in eq. (5.1) (see [21] for a detailed analysis). Under suitable scaling of the ξ field, one can get rid of the constant, thereby restoring the canonical form of the kinetic term of ξ. In the process, however, it only leads to another constant parameter b being invoked in the $\xi F^{\star} F$ coupling term in the string effective action as in (5.1).

The introduction of both the parameters a and b may also be motivated from another interesting viewpoint. For specific values of a and b, the entire action (5.1) is shown [16] to lead exactly to the field equations corresponding to a four dimensional effective compactified version of a higher dimensional (bulk) Einstein-Maxwell-dilaton-Kalb-Ramond theory in Randall-Sundrum (RS) scenario [14] (that provides a resolution to the well-known Planck-electroweak hierarchy problem). The standard Kaluza-Klien decomposition of all the bulk fields gives rise to both massless and towers of massive projections on the four dimensional subspace — the '3-brane' — in which all the standard model fields are supposed to be confined. Demanding that all the massive modes must lie in the TeV range in a RS picture, it has been found that the massless modes of both the bulk KR field [37, 38, 15] and the scalar dilaton [16] are suppressed by a huge exponential (RS 'warp') factor on the 3-brane, while the massless mode of the gauge field suffers no such suppression [39]. The net result in a low energy effective four dimensional theory is that the exponential dilaton coupling with the KR field and the gauge field is virtually washed out, while in contrast, the strength of the KR-electromagnetic interaction, via the Chern-Simons extension, is immensely boosted by the large warp factor. Expressing the KR massless mode in terms of the axion, one finds that the four dimensional effective action corresponds exactly with (5.1) for $a \sim e^{-\sigma} \ll 1$ and $b \sim e^{\sigma} \gg 1$ [16], where σ is the RS warping.

Thus explaining the possible origin of the constant parameters a and b and their respective values in different circumstances, let us now look for complete black hole descriptions of the solutions of field equations in various situations in the context of dilaton-axion coupled Einstein-Maxwell theory. For convenience, we choose to work in the natural units setting $2\kappa = 16\pi G = 1$.

A comparison of the action (5.1) with the original action (2.1) reveals the following coupling specifications:

$$\omega(\phi) = e^{2a\phi}\,; \qquad \alpha(\phi) = e^{-a\phi}\,; \qquad \beta(\xi) = b\xi \tag{5.4}$$

and shows that the set of equations (2.21), given in section 2, now take the form

$$\gamma = 4q_m^2 e^{-a\phi} + 4\left(q_e - q_m b\xi\right)^2 e^{a\phi} \tag{5.5}$$

$$\psi'^2 = \phi'^2 + e^{2a\phi}\xi'^2 \tag{5.6}$$

$$4\; q_m b\left(q_e - q_m b\xi\right) e^{a\phi} = -R^2\left[\left(r - r_+\right)\left(r - r_-\right) e^{2a\phi}\;\xi'\right]'. \tag{5.7}$$

In what follows, we carefully examine the compatibility of the general expressions for the metric functions given in sections 4.1 and 4.2 for AF and AN spacetimes and the corresponding expressions for the effective scalar and the effective coupling (ψ and γ respectively), with the above set of equations (5.5). Resorting separately to the AF and AN cases, we try to determine the functional forms of the scalars ϕ and ξ by solving eqs. (5.5) in some specific situations corresponding to some typical values of the parameters a and b.

5.1 Asymptotically flat black holes

We list below the full set of solutions obtained in section 4.1 for the metric, the effective scalar ψ and the effective coupling γ, as well as the expressions for the electromagnetic field components, given in section 2.

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Line element:

$$ds^2 = -\frac{(r-r_+)(r-r_-)}{(r-r_0)^{2-2n}(r+r_0)^{2n}}dt^2 + \frac{(r-r_0)^{2-2n}(r+r_0)^{2n}}{(r-r_+)(r-r_-)}dr^2 + \\ + \frac{(r+r_0)^{2n}}{(r-r_0)^{2n-2}}d\Omega^2 \,. \tag{5.8}$$

Effective scalar:

$$\psi(r) = \psi_0 + 2\sqrt{n(1-n)}\ \ln\left(\frac{r-r_0}{r+r_0}\right). \tag{5.9}$$

Effective coupling:

$$\gamma(r) = K_1\left(\frac{r-r_0}{r+r_0}\right)^{2(1-n)} +\ K_2\left(\frac{r-r_0}{r+r_0}\right)^{-2n}. \tag{5.10}$$

Electromagnetic field components:

$$F_{\mu\nu} : F_{\rm tr} = \frac{(q_e - q_m b\ \xi)\, e^{a\phi}}{(r-r_0)^{2(1-n)}(r+r_0)^{2n}}dt\wedge dr\,; \qquad F_{\vartheta\varphi} = q_m \sin\vartheta d\vartheta\wedge d\varphi \tag{5.11}$$

where

$$r_\pm = m_0 \pm \sqrt{m_0^2 + r_0^2 - \frac{1}{8}\left(\frac{K_1}{n} + \frac{K_2}{1-n}\right)}; \\ r_0 = \frac{1}{16m_0}\left(\frac{K_1}{n} - \frac{K_2}{1-n}\right); \qquad m_0 = m - (2n-1)r_0\,. \tag{5.12}$$

The relationships of $\{\psi, \gamma\}$ with $\{\phi, \xi\}$ are given by eqs. (5.5a) and (5.5b). Letting the scalar fields ϕ and ξ approach the limiting values ϕ_0 and ξ_0, as $r\to\infty$, in an asymptotically flat (AF) spacetime, it is possible to relate the scalar-shielded electric and magnetic charges q_e and q_m to the total (bare) electric and magnetic charges Q_e and Q_m. These are defined through the relations [25]

$$Q_e = -\frac{1}{4\pi}\int_S^\star F = -\frac{1}{4\pi}\int_S E_r v^r dS\,; \qquad Q_m = -\frac{1}{4\pi}\int_S F = \frac{1}{4\pi}\int_S B_r v^r dS \tag{5.13}$$

where the two-sphere S is the boundary of a three-dimensional hypersurface Σ, pushed to spatial infinity; and the integrals are of the normal (radial) components of the electric and magnetic vectors $E_\alpha = F_{\alpha\beta}n^\beta$ and $B_\alpha = -{}^\star F_{\alpha\beta}n^\beta$ respectively, on S; n^μ being a unit normal to Σ. Using the expressions (2.8) for $F_{\rm rt}$ and $F_{\vartheta\varphi}$ given in section 2 and the expressions (5.4) for the couplings ω, α and β, one finds in the asymptotic limit:

$$Q_e = (q_e - q_m b\ \xi_0)\, e^{a\phi_0}\ ; \qquad Q_m = q_m \tag{5.14}$$

whence

$$F_{\rm tr} = \frac{\left[Q_e e^{-a\phi_0} - Q_m b\,(\xi - \xi_0)\right] e^{a\phi}}{(r-r_0)^{2(1-n)}(r+r_0)^{2n}}\ dt\wedge dr\,; \qquad F_{\vartheta\varphi} = Q_m \sin\vartheta d\vartheta\wedge d\varphi\,. \tag{5.15}$$

In fact it is well-known in the usual string theoretic scenario (corresponding to $a = b = 1$) that a non-vanishing ξ_0 is of no consequence by virtue of the Peccei-Quinn (PQ) shift symmetry of the axion. However, the PQ shift being a classical symmetry of the heterotic action, it can be broken by instantons in a non-perturbative approach to string theory. We, therefore, retain the constant ξ_0 wherever it is admissible, although it has no significant role in our analysis of black holes in presence of dilaton and axion.

Using eqs. (5.14) and the two expressions (5.5a) and (5.10) for the coupling function γ, we find in the asymptotic limit

$$K_1 + K_2 = 4q_m^2 e^{-a\phi_0} + 4\left(q_e - q_m b\xi_0\right)^2 e^{a\phi_0} = 4\left(Q_m^2 + Q_e^2\right) e^{-a\phi_0}\,. \tag{5.16}$$

The set of equations (5.5) take the form

$$Q_m^2 e^{-a\phi} + \left[Q_e e^{-a\phi_0} - Q_m b\left(\xi - \xi_0\right)\right]^2 e^{a\phi} = \frac{Q^2\left(r - r_0\right)^2 + \ K_2 r r_0}{(r + r_0)^{2(1-n)}(r - r_0)^{2n}} \tag{5.17}$$

$$\phi'^2 + e^{2c\phi}\xi'^2 = \frac{16 r_0^2 n\ (1 - p)}{(r^2 - r_0^2)^2} \tag{5.18}$$

$$4Q_m b\left[Q_e e^{-a\phi_0} - Q_m b\left(\xi - \xi_0\right)\right] e^{a\phi} = -\frac{\left[\left(r - r_+\right)\left(r - r_-\right) e^{2a\phi}\ \xi'\right]'}{(r + r_0)^{-2n}(r - r_0)^{2(n-1)}}\,. \tag{5.19}$$

where $Q^2 \equiv Q_e^2 + Q_m^2$.

In order to determine the constant r_0 in terms of the mass m, charges Q_e and Q_m, we need to fix atleast one of the parameters a and b so that the eqs. (5.17) are satisfied. In the process the values of the other undetermined parameters n and K_2 could also be fixed up. (The constant K_1 is of course given in terms of Q_m, Q_e, ϕ_0 and K by virtue of eq. (5.16)). This is a very convenient way of deriving complete solutions of the field equations corresponding to the action (5.1). We consider first the special case $|b| = |a|$, which evidently includes the usual string scenario where a and b are both equal to unity, and then look into the more general choice $|b| \neq |a|$ and resort to a particular case which is relevant to string theory.

Case $|b| = |a|$. In this case the set of equations (5.17) can be satisfied uniquely for $n = 1/(1 + a^2)$ and $K_2 = 0$, whence referring to the expressions (5.12) one finds

$$r_0 = \frac{(1 + a^2) Q^2 e^{-a\phi_0}}{4m_0}\,; \qquad m_0 = m - \left(\frac{1 - a^2}{1 + a^2}\right) r_0\,, \qquad \left(Q^2 = Q_e^2 + Q_m^2\right) \tag{5.20}$$

and

$$r_+ = 2m_0 - r_0\,, \qquad r_- = r_0\,. \tag{5.21}$$

The metric components g_{tt} $(= g^{rr})$ and $g_{\vartheta\vartheta}$ $(= g_{\varphi\varphi}/\sin^2\vartheta)$ then take the form

$$g_{\rm tt}(r) = -\left(\frac{r + r_0 - 2m_0}{r + r_0}\right)\left(\frac{r - r_0}{r + r_0}\right)^{\frac{1-a^2}{1+a^2}}\,, \qquad g_{\vartheta\vartheta}(r) = (r + r_0)^2\left(\frac{r - r_0}{r + r_0}\right)^{\frac{2a^2}{1+a^2}}\,. \tag{5.22}$$

By making a coordinate shift $r + r_0 \to r$, the whole set of solutions can finally be expressed as

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{2m_0}{r}\right)\left(1-\frac{2r_0}{r}\right)^{\frac{1-a^2}{1+a^2}} dt^2 + \\
&\quad + \left(1-\frac{2m_0}{r}\right)^{-1}\left(1-\frac{2r_0}{r}\right)^{\frac{a^2-1}{a^2+1}} dr^2 + \\
&\quad + r^2\left(1-\frac{2r_0}{r}\right)^{\frac{2a^2}{1+a^2}} d\Omega^2 \qquad (5.23)
\end{aligned}
$$

$$
\phi(r) = \phi_0 + \frac{1}{a}\ln\left[\frac{1}{Q^2}\left(1-\frac{2r_0}{r}\right)^{\frac{-2a^2}{1+a^2}}\left\{Q_m^2 + Q_e^2\left(1-\frac{2r_0}{r}\right)^{\frac{4a^2}{1+a^2}}\right\}\right] \qquad (5.24)
$$

$$
\begin{aligned}
\xi(r) &= \xi_0 + \qquad (5.25)\\
&\quad + \frac{Q_m Q_e}{a} e^{-a\phi_0}\left[1-\left(1-\frac{2r_0}{r}\right)^{\frac{4a^2}{1+a^2}}\right]\left[Q_m^2 + Q_e^2\left(1-\frac{2r_0}{r}\right)^{\frac{4a^2}{1+a^2}}\right]^{-1}
\end{aligned}
$$

$$
F_{\mu\nu} \;:\; F_{\rm tr} = \frac{Q_e}{r^2}\, dt\wedge dr\,; \qquad F_{\vartheta\varphi} = Q_m \sin\vartheta\; d\vartheta\wedge d\varphi \qquad (5.26)
$$

where r_0 and m_0 are given in eqs. (5.20).

These solutions represent a regular black hole horizon at $r = 2m_0$ and a curvature singularity at $r = 2r_0$. One may now define the total (bare) dilaton and axion charges (Q_ϕ and Q_ξ respectively) in a way similar to the total (bare) electromagnetic charges Q_e and Q_m in an asymptotically flat spacetime:

$$
\begin{aligned}
Q_\phi &= -\frac{1}{4\pi}\int_S^* d\phi = \frac{a\left(Q_m^2 - Q_e^2\right)e^{-a\phi_0}}{m_0} \\
Q_\xi &= -\frac{1}{4\pi}\int_S^* d\xi = \frac{2a\; Q_e Q_m}{m_0}\; e^{-2a\phi_0}\,. \qquad (5.27)
\end{aligned}
$$

For either Q_e or Q_m equal to zero the axion charge vanishes, while the dilaton charge flips a sign. This follows from the fact that a magnetically charged ($Q_e = 0, Q_m = Q$) dilaton black hole transforms to an electrically charged ($Q_m = 0, Q_e = Q$) dilaton black hole, or vice versa. In effect, this is a generalization of electric-magnetic duality symmetry in presence of the dilaton field [5]-[31].

A few special/limiting cases are in order:

(i) $a = 1$: As mentioned earlier, this case corresponds to the bosonic sector of the ten dimensional heterotic superstring toroidally compactified to four spacetime dimensions. The above solutions for the spacetime line element and dilaton and axion reduce respectively to

$$
ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(1-\frac{2m}{r}\right)^{-1} dr^2 + r\,(r-2r_0)\,d\Omega^2 \qquad (5.28)
$$

$$\phi(r) = \phi_0 + \ln\left[\frac{Q^2 r^2 - 4Q_e^2 r_0 (r - r_0)}{Q^2 r (r - 2r_0)}\right];$$

$$\xi(r) = \xi_0 + \left[\frac{4Q_m Q_e e^{-\phi_0} r_0 (r - r_0)}{Q^2 r^2 - 4Q_e^2 r_0 (r - r_0)}\right] \tag{5.29}$$

where $r_0 = Q^2 e^{-\phi_0}/(2m)$. The horizon is located at $r = 2m$ and there is a curvature singularity at $r = 2r_0$.

For $Q_e = 0, Q_m = Q$ (or, $Q_m = 0, Q_e = Q$), the solutions correspond to the magnetically (or, electrically) charged dilaton black hole found independently by Garfinkle, Horowitz and Strominger (GHS) [40] and by Gibbons [5] (which has also been explained in the works of Gibbons and Maeda (GM) [36]). Accordingly, such a black hole is referred to as a GHS black hole, or sometimes a GMGHS black hole. Whereas a number of characteristically different dilaton black hole solutions in arbitrary spacetime dimensions and for arbitrary values of the parameter a have been illustrated by GM [36], GHS [40] elegantly obtained a singly charged (i.e., for either $Q_e = 0$ or $Q_m = 0$) dilaton black hole in the string case $a = 1$ only. However, the works of both GHS and GM have been based on the presumption that the KR axion field is zero (or, atleast trivial), which of course follows automatically when one considers a singly charged configuration. As can seen readily from the above expressions (5.28) and (5.29), the $Q_m = 0, Q_e = Q$ GMGHS configuration merely produces a mirror image of the solutions corresponding to $Q_e = 0, Q_m = Q$ under the transformation $\phi \to -\phi$.

Non-trivial dilaton-axion configurations can be generated from the magnetically (or, electrically) charged GMGHS dilaton black hole solution (with trivial axion), by utilizing the invariance of the complete set of field equations under the full symmetry group, viz., the $\mathrm{SL}(2, \mathbb{R})$ group [29, 31]. Such a symmetry group is generated by repeated applications of the Peccei-Quinn shift of the axion, viz., $\xi \to \xi + p$, and the duality $\lambda \to -1/\lambda$, where p is a constant and $\lambda \equiv \xi + ie^{-\phi}$ is a complex scalar constructed out of the dilaton and the axion. The solution generation process is accomplished on stipulating the gauge field strength to transform under the $\mathrm{SL}(2, \mathbb{R})$ transformations as $F_+^{\mu\nu} \to -\lambda F^{\mu\nu}$, $F_-^{\mu\nu} \to -\bar{\lambda}^\star F^{\mu\nu}$, where $F_\pm^{\mu\nu} = F^{\mu\nu} \pm i^\star F^{\mu\nu}$ and $\bar{\lambda}$ is the complex conjugate of the so-called 'axidilaton' field λ. For $\xi = 0$, the transformation $\lambda \to -1/\lambda$, simply implies $\phi \to -\phi$. Although the non-trivial dilaton and axion field configurations which have been found in [29] by utilizing the duality invariance are not explicitly in the same form as given above, it can be shown easily that the solutions (5.28), (5.29) indeed result from application of the $\mathrm{SL}(2, \mathbb{R})$ transformations on the magnetically (or, electrically) charged GMGHS dilaton black hole. In fact, it can be shown more rigorously that a non-trivial dilaton-axion configuration can be generated from the magnetically (or, electrically) charged dilaton black hole configuration by utilizing the $\mathrm{SL}(2, \mathbb{R})$ invariance, even when the parameter a is not necessarily equal to unity [41].

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(ii) $a \ll 1$: For small values of the parameter a, the solutions (5.24) for the dilaton and the axion can be expanded as

$$\phi(r) = \phi_0 + \frac{4ar_0}{r}\left(\frac{Q_m^2 - Q_e^2}{Q^2}\right) + \mathcal{O}(a^3)\,; \qquad \xi(r) = \xi_0 + \frac{4ar_0}{r}\left(\frac{Q_m Q_e}{Q^2}\right) + \mathcal{O}(a^3) \tag{5.30}$$

where $Q^2 = Q_m^2 + Q_e^2$ and $r_0 = Q^2/(4m_0) + \mathcal{O}(a^2)$. Eqs. (5.20) can be solved readily to obtain

$$r_0 = \frac{1}{2}\left(m - \sqrt{m^2 - Q^2}\right) + \mathcal{O}(a^2)\,, \qquad m_0 = \frac{1}{2}\left(m - \sqrt{m^2 - Q^2}\right) + \mathcal{O}(a^2)\,. \tag{5.31}$$

In the limit $a \to 0$, eqs. (5.30) imply that both the dilaton and the axion become trivial and one has the familiar Einstein-Maxwell system. The line element (5.23) takes the form of the standard dyonic Reissner-Nordström (RN) black hole with horizons at $r = 2m_0 = m + \sqrt{m^2 - Q^2}$ and at $r = 2r_0 = m - \sqrt{m^2 - Q^2}$.

(iii) $a \gg 1$: For large values of the parameter a, the equations (5.20) can be approximated as

$$r_0 \approx \frac{a^2 Q^2 e^{-a\phi_0}}{4m_0}\,, \qquad m_0 \approx m + r_0\,. \tag{5.32}$$

Assuming that ϕ_0 is non-zero and reasonably larger that of $\sim \mathcal{O}(1/a)$ for large a, one may consider that the constant r_0 (and hence m_0) does not become arbitrarily larger than m in the limit $a \to \infty$. Retaining only the terms upto the leading order in $1/a$, the solution for the spacetime line element, as well as the solutions for the dilaton ϕ and the axion ξ can approximately be given as

$$ds^2 \approx -\left(1 - \frac{2m}{r - 2r_0}\right)dt^2 + \left(1 - \frac{2m}{r - 2r_0}\right)^{-1} dr^2 + (r - 2r_0)^2\, d\Omega^2 \tag{5.33}$$

$$\phi(r) \approx \phi_0 + \frac{1}{a}\ln\left[\frac{1}{Q^2}\left(1 - \frac{2r_0}{r}\right)^{-2}\left\{Q_m^2 + Q_e^2\left(1 - \frac{2r_0}{r}\right)^4\right\}\right] \tag{5.34}$$

$$\xi(r) \approx \xi_0 + \frac{Q_m Q_e}{a}\, e^{-a\phi_0}\left[1 - \left(1 - \frac{2r_0}{r}\right)^4\right]\left[Q_m^2 + Q_e^2\left(1 - \frac{2r_0}{r}\right)^4\right]^{-1} \tag{5.35}$$

Of course, one may also neglect the second term in the expression for ξ which contains a suppression factor of $(1/a)e^{-a\phi_0}$ with respect to ξ_0. By making a coordinate transformation $r - 2r_0 \to r$, we find that the resulting approximate expression for the line element reduces to the standard Schwarzschild one. However the dilaton, axion and the U(1) gauge field have non-vanishing solutions given by,

$$\phi(r) \approx \phi_0 + +\frac{1}{a}\ln\left[\frac{Q_m^2}{Q^2}\left(1 - \frac{2r_0}{r}\right)^2 + +\frac{Q_e^2}{Q^2}\left(1 - \frac{2r_0}{r}\right)^{-2}\right]\,;$$

$$\xi(r) = \xi_0 + \mathcal{O}\left(\frac{e^{-a\phi_0}}{a}\right) \tag{5.36}$$

$$F_{\mu\nu} \ : \ F_{\rm tr} = \frac{Q_e}{r^2} dt \wedge dr \, ; \qquad F_{\vartheta\varphi} = Q_m \sin\vartheta d\vartheta \wedge d\varphi \tag{5.37}$$

Solving eqs. (5.32) one finds two possible values of r_0 given by $-\frac{1}{2}\big(m \pm \sqrt{m^2 + a^2 Q^2 e^{-a\phi_0}}\big)$ which imply two possible limiting configurations of the dilaton (and also the axion) for large a. In the limit $a \to \infty$, we have the Schwarzschild line element with both dilaton and axion fields becoming trivial. This is of course quite expected since under the presumption that for large a, ϕ_0 is reasonably larger that of $\sim \mathcal{O}(1/a)$, so that the $e^{-a\phi} F^2$ coupling term in the action virtually disappears. What remains is the coupling $a\xi F^\star F$ in the action, and ξ being trivial (as is ϕ) in the limit $a \to \infty$, the $F^\star F$ term as usual does not produce any change to the vacuum solution (which is the Schwarzschild solution) of the field equations corresponding to the pure general relativistic action.[3]

Case $|b| \neq |a|$. When the parameters b and a are unequal, it is not always possible to obtain an analytic closed form black hole solution following the our metric ansatz. However, some specific choices of values of b and a (where $|b| \neq |a|$), enable one to fix up the values of the three undetermined constant parameters r_0, n and K_2 in terms of the physical constants m, Q_e, Q_m and ϕ_0 (and may also ξ_0) so that all the field equations (5.17) are satisfied. We consider the case $b \ll 1$ and $a = 1$, which is particularly relevant for string theory and can be motivated from two different viewpoints:

- Firstly, the action in this case closely resembles to the usual dimensionally reduced action (5.3) for the ten dimensional heterotic string apart from the Planck mass suppressed gauge Chern-Simons term appearing in the definition of the modified Kalb-Ramond (KR) three form $H_{\mu\nu\lambda}$ (see eq. (5.2)). This term can be neglected in the leading order which amounts to neglecting the Planck mass suppressed coupling term $\xi F^\star F$ that appears in the general action (5.1) by virtue of the KR-electromagnetic interaction via the Chern-Simons term. This, in turn, implies that we are considering the axion field ξ to be source-free.
- Secondly, the choice of the parameters $a = 1, b \ll 1$, has a direct correspondence with an exact low energy string setting (retaining the Chern-Simons term) which contains a number of unbroken U(1) subgroups belonging to a large gauge group. Such a scenario has already been discussed in detail in [32, 33].

For $b \ll 1, a = 1$, eq. (5.17c) yields the general solution for the axion in the form of a quadrature:

$$\xi(r) = \xi_0 + \xi_1 \int^r \frac{e^{-2\phi(r')} dr'}{(r' - r_+)(r' - r_-)} + \mathcal{O}(b) \tag{5.38}$$

[3]It is well-known that the addition of a $\int d^4x \sqrt{-g} F_{\mu\nu} {}^\star F^{\mu\nu}$ term to the pure Einstein action $\int d^4x \sqrt{-g} R$ does not alter the vacuum Einstein's equations $R_{\mu\nu} = 0$ and hence the Schwarzschild solution.

where ξ_0 and ξ_1 are constants. However, a closer inspection reveals that exact solutions of eqs. (5.17a) and (5.17b) determines the metric as well as the dilaton only when $\xi_1 = 0$. This implies that the axion field ξ is trivial upto $\mathcal{O}(b)$. In other words, a complete solution of the full set of equations (5.17) are guaranteed only when the axion has a negligible effect in shaping the spacetime geometry. Now, the solutions for the line element and the dilaton ϕ correspond exactly to the charged dyonic black hole [5, 36] in four dimensional low energy effective string theory, if the electromagnetic charges Q_e and Q_m (or, q_e and q_m) are associated with two different U(1) gauge fields with field strengths $F^i_{\mu\nu}$, $(i = 1, 2)$ [4, 36, 32, 33, 42]. In such case the axion ξ becomes automatically source-free and as such the axion charge Q_ξ vanishes. The source-free property of the axion is retained in a further generalization to the scenario in which either U(1) fields are due to both electric and magnetic monopole configurations (with respective charges Q^i_e and Q^i_m, $(i = 1, 2)$) [32]. If there are N number of electric and magnetic charges Q^i_e and Q^i_m $(i = 1, \ldots, N)$ associated with N different U(1) gauge fields $F^i_{\mu\nu}$ $(i = 1, \ldots, N)$, then it has been shown that [33] an exact dyonic black hole solution exists only when $\sum Q^i_e Q^i_m = 0$, which in turn implies that the term $\sum \xi F^{i\,\star} F^i$ in the effective action is no longer relevant. Incidentally, the ξ field turns out to be trivial and the solution is similar to the dyonic dilaton black hole described in [5, 36].

Neglecting the terms of $\mathcal{O}(b)$ and considering $\xi_1 = 0$ in the above equation (5.38), we write down the full set of solutions of eqs. (5.17). The solutions are given for $n = 1/2$ and $K_2 = 4Q_m^2 e^{-\phi_0}$ (or $K_2 = 4Q_e^2 e^{-\phi_0}$ — the solutions are invariant under the interchange $Q_e \leftrightarrow Q_m$):

$$\begin{aligned} ds^2 &= -\frac{(r - r_+)(r - r_-)}{r^2 - r_0^2} dt^2 + \frac{r^2 - r_0^2}{(r - r_+)(r - r_-)} dr^2 + \\ &\quad + \left(r^2 - r_0^2\right) d\Omega^2 \end{aligned} \tag{5.39}$$

$$\phi(r) = \phi_0 + \ln\left(\frac{r - r_0}{r + r_0}\right); \qquad \xi(r) = \xi_0 \tag{5.40}$$

$$F_{\mu\nu} : F_{\rm tr} = \frac{Q_e}{(r + r_0)^2} dt \wedge dr\,; \qquad F_{\vartheta\varphi} = Q_m \sin\vartheta\, d\vartheta \wedge d\varphi \tag{5.41}$$

where

$$r_0 = \frac{\left(Q_e^2 - Q_m^2\right) e^{-\phi_0}}{2m}; \quad \text{and} \quad r_\pm = m \pm \sqrt{m^2 + r_0^2 - \left(Q_e^2 + Q_m^2\right) e^{-\phi_0}}\,. \tag{5.42}$$

There are regular horizons located at $r = r_\pm$ and a curvature singularity appears at $r = r_0$. The dilaton charge turns out to be twice the constant r_0 in magnitude:

$$Q_\phi = \frac{\left(Q_m^2 - Q_e^2\right) e^{-\phi_0}}{m}\,. \tag{5.43}$$

For either Q_e or Q_m equal to zero, the above solutions reduce respectively to the GHS magnetically or electrically charged black hole (of course, under the coordinate shift $r + r_0 \to r$), while in the special cases $Q_e = +Q_m$ or $-Q_m$, the dilaton charge vanishes and one gets back the standard Reissner-Nordström black hole.

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5.2 Asymptotically non-flat black holes

We recall the full set of general expressions derived in section 4.2 for the asymptotically non-flat (AN) spacetime metric, the effective scalar ψ, the effective coupling γ, as well as the expressions for the electromagnetic field components, obtained in section 2. We summarize below the essential results:

Line element:

$$ds^2 = -\frac{(r-r_+)\,(r-r_-)}{r^2\,(2r_0/r)^{2n}}\,dt^2 + \frac{r^2\,(2r_0/r)^{2n}}{(r-r_+)\,(r-r_-)}\,dr^2 + r^2\left(\frac{2r_0}{r}\right)^{2n} d\Omega^2\,. \quad (5.44)$$

Effective scalar:

$$\psi(r) = \psi_0 - 2\sqrt{n(1-n)}\,\ln\left(\frac{2r_0}{r}\right). \quad (5.45)$$

Effective coupling:

$$\gamma(r) = \left(4nr^2 + K_2\right)\left(\frac{2r_0}{r}\right)^{2n}. \quad (5.46)$$

Electromagnetic field components:

$$F_{\mu\nu} : F_{\rm tr} = \frac{(q_e - q_m b\xi)\,e^{a\phi}}{r^2\,(2r_0/r)^{2n}} dt\wedge dr\,; \qquad F_{\vartheta\varphi} = q_m \sin\vartheta d\vartheta\wedge d\varphi \quad (5.47)$$

where

$$r_\pm = \left(\frac{1}{1-n}\right)\left[m \pm \sqrt{m^2 - (1-n)\frac{K_2}{4}}\right]. \quad (5.48)$$

The set of equations (5.5) in this case take the form

$$\begin{aligned}
&(a)\quad q_m^2 e^{-a\phi} + (q_e - q_m b\,\xi)^2\,e^{a\phi} = \left(nr^2 + \frac{K_2}{4}\right)\left(\frac{2r_0}{r}\right)^{2n} \\
&(b)\quad \phi'^2 + e^{2a\phi}\xi'^2 = \frac{4n(1-n)}{r^2} \\
&(c)\quad 4q_m b\,(q_e - q_m b\,\xi)\,e^{a\phi} = -r^2\left(\frac{2r_0}{r}\right)^{2n}\left[(r-r_+)\,(r-r_-)\,e^{2a\phi}\xi'\right]'.
\end{aligned} \quad (5.49)$$

We again need to determine the values of the arbitrary constant K_2, for which the above equations (5.49) are satisfied for appropriate choice of the values of the parameters a and b. The values of the other undetermined parameter n can also be fixed up in the process. As before, we resort to the special case $|b| = |a|$ first (which includes the usual string case $b = a = 1$) and then the more general situation $|b| \neq |a|$. For the latter choice we shall focus to a particular case relevant to string theory.

Case $|b| = |a|$**.** In this case, the solutions of the set of equations (5.49) are given uniquely for $n = 1/(1 + a^2)$ and $K_2 = 0$, whence referring to the expressions (5.48) one finds $r_+ = 2m/(1 + n)$ and $r_- = 0$. The full set of solutions can therefore be expressed as

$$\begin{aligned} ds^2 &= -\left(\frac{r}{2r_0}\right)^{2n}\left[1 - \frac{2m}{(1-n)r}\right]dt^2 + \\ &\quad + \left(\frac{2r_0}{r}\right)^{2n}\left[1 - \frac{2m}{(1-n)r}\right]^{-1} dr^2 + r^2\left(\frac{2r_0}{r}\right)^{2n} d\Omega^2 \end{aligned} \tag{5.50}$$

$$\phi(r) = \frac{1}{a}\ln\left[\frac{q_m^2 q^4 + q_e^2 n^2 (2r_0)^{4n} r^{4(1-n)}}{q^4 n (2r_0)^{2n} r^{2(1-n)}}\right] \tag{5.51}$$

$$\xi(r) = \frac{q_m q_e}{a}\left[\frac{q^4 - n^2 (2r_0)^{4n} r^{4(1-n)}}{q_m^2 q^4 + q_e^2 n^2 (2r_0)^{4n} r^{4(1-n)}}\right] \tag{5.52}$$

$$F_{\mu\nu} \;:\; F_{\rm tr} = \frac{nq_e}{q^2} dt \wedge dr\,; \qquad F_{\vartheta\varphi} = q_m \sin\vartheta d\vartheta \wedge d\varphi \tag{5.53}$$

where $q^2 = q_e^2 + q_m^2$.

The constant r_0 has dimension of r and the event horizon at $r = 2m/(1 - n)$ is regular, i.e., all the scalar invariants, viz., Ricci, squared Ricci and Kretschmann, are well behaved on that surface; $r = 0$ of course is a physical singularity. The causal structure is similar to the standard Schwarzschild black hole and $r = 2m/(1 - n)$ is a null hypersurface. However, as $r \to \infty$, the metric as well as the dilaton and axion fields ϕ and ξ are divergent, thus showing the asymptotically non-flat nature of the spacetime. If the values of either q_e or q_m changes, the metric remains invariant since it does not involve the gauge charges explicitly. This also implies that the extremal limit is absolutely redundant for the above line element. The solutions for the dilaton and the axion, however, depend on q_e and q_m and vary with either of them. The axion in fact vanishes for either of q_e or q_m equals to zero, and also for $r = (q/\sqrt{n})^{1/(1-n)}(2r_0)^{-n/(1-n)}$ even when both the gauge charges are non-vanishing.

Although, the metric above is independent of the electromagnetic charges q_e and q_m, it may still be looked upon as the one that represents a charged black hole. This is because, not only the metric signifies a dilaton-axion coupled gravito-electromagnetic field configuration, but also the fact that in the usual string theoretic scenario ($a = 1, n = 1/2$) the string frame metric $\tilde{g}_{\mu\nu}$ which is related to the Einstein frame metric $g_{\mu\nu}$ via the conformal transformation $\tilde{g}_{\mu\nu} = e^{\phi} g_{\mu\nu}$, indeed involves q_e and q_m through the solution of the conformal factor e^{ϕ}. In the string frame, the string length scale l_s is taken as the fundamental unit for all physical measurements, while the string coupling g_s is determined by the dilaton through the relation $g_s^2 = e^{\phi}$. Thus the effective Planck length, i.e., the coefficient of the string frame scalar curvature, varies with the dilaton. As such, there is no usual way to realize any experimental data physically in the string frame since the Planck length has a direct bearing upon macroscopic physics through the strength of gravity. The difficulty is overcome in the Einstein frame in which the Planck length being a universal constant, is used as a fundamental unit. This is the reason why the Einstein frame is always preferred and we have concentrated on deriving our solutions in this frame rather than in the string frame for most part of the present paper. For a recent discussion on the comparative studies in string and Einstein frames, see [43].

In the present case, however, we shall also consider the string frame results (see below in the discussion for the special case $a = 1$). The solutions for the spacetime line element as well the dilaton and the axion in a few special/limiting cases for some specific values of the parameter a are given as follows:

(i) $a = 1$: The above solution (5.50) for the (Einstein frame) line element reduces to

$$ds^2 = -\left(\frac{r-4m}{2r_0}\right) dt^2 + \left(\frac{2r_0}{r-4m}\right) dr^2 + 2r_0 r d\Omega^2 \tag{5.54}$$

while the solutions (5.51) for dilaton and axion take the form

$$\phi(r) = \ln\left(\frac{q_m^2 q^4 + q_e^2 r_0^2 r^2}{q^4 r_0 r}\right); \qquad \xi(r) = \frac{q_e q_m \left(q^4 - r_0^2 r^2\right)}{q_m^2 q^4 + q_e^2 r_0^2 r^2}. \tag{5.55}$$

The components of the electromagnetic field strength also change: $F_{\rm tr} = q_e/(2q^2) dt \wedge dr$; $F_{\vartheta\varphi} = q_m \sin\vartheta d\vartheta \wedge d\varphi$. The event horizon is at $r = 4m$ and $r = 0$ is the curvature singularity.

After a coordinate redefinition $\rho = \sqrt{2} r_0 r/q$ the string frame line element as well as the solutions for dilaton and axion can be expressed in an elegant way as

$$\begin{aligned} d\tilde{s}^2 = & \left(\frac{q_e^2}{q^2} + \frac{2q_m^2}{\rho^2}\right)\left[-\frac{\rho^2}{4r_0^2}\left(1 - \frac{4\sqrt{2} r_0 m}{q\rho}\right) dt^2 + \right. \\ & \left. + \left(1 - \frac{4\sqrt{2} r_0 m}{q\rho}\right)^{-1} d\rho^2 + \rho^2 d\Omega^2\right] \end{aligned} \tag{5.56}$$

$$\phi(r) = \ln\left(\frac{2q_m^2 q^2 + q_e^2 \rho^2}{\sqrt{2} q^3 \rho}\right); \qquad \xi(r) = \frac{q_e q_m \left(2q^2 - \rho^2\right)}{2q_m^2 q^2 + q_e^2 \rho^2} \tag{5.57}$$

where as before $q^2 = q_e^2 + q_m^2$ and r_0 now has dimension of square root of the redefined coordinate ρ. The event horizon is at $\rho = 4\sqrt{2}\, r_0 m/q$ and unlike asymptotically flat spacetimes, the string loop corrections at large ρ are of much significance here, as the string coupling $e^{\phi} \sim \rho$ in the asymptotic limit as long as q_e is non-zero.

The special cases corresponding to $q_e = 0$ or $q_m = 0$ are of considerable interest. In either of the instances, the axion field ξ is null and we have magnetically or electrically charged dilaton black holes with curved asymptotes. There have been detailed studies on the characteristic properties and thermodynamics of such special (dilaton only) black hole solutions, found originally by Chan et al. [7], in both Einstein and string frames.[4] However, the general (dyonic) black

[4]Asymptotically non-flat magnetically charged dilaton black hole solution for a particular value of the mass m and magnetic charge q_m have also been obtained in [44].

hole solutions in either frames, in presence of both non-trivial dilaton and non-trivial axion fields, as shown respectively in (5.54), (5.55) and (5.56), (5.57) (as well as those given by (5.50) - (5.53) in Einstein frame for arbitrary value of the parameter a), are entirely new findings in the present work.

For $q_e = 0$ and $q_m = q$, the solution for the dilaton is $\phi = \ln(q^2/(r_0 r)) = \ln(\sqrt{2}q/\rho)$. This solution flips a sign for the dual choice $q_m = 0$ and $q_e = q$. The Einstein metric is obviously invariant since it involves neither q_e nor q_m explicitly. Thus the electric-magnetic duality, which has been generalized here in presence of the dilaton, happens to be a symmetry of nature, as it should be, regardless of the asymptotic properties of the background spacetime. However, it should be mentioned here that one cannot, in principle, make allusion to (asymptotic) bare electromagnetic charges Q_e, Q_m in spacetimes that are asymptotically curved, but rather insist upon the isolated charges q_e, q_m. It is also not possible to define in the usual way a (bare) dilaton charge Q_ϕ (nor a bare axion charge Q_ξ) in such spacetimes.

In the string frame, however, the metric certainly changes when one switches over from the case $q_e = 0, q_m = q$ to the case $q_m = 0, q_e = q$, whence the dilaton ϕ flips a sign. In the latter case, viz, $q_m = 0, q_e = q$, the line element given by

$$d\tilde{s}_e^2 = -\frac{\rho^2}{4r_0^2}\left(1 - \frac{4\sqrt{2}r_0 m}{q\rho}\right)dt^2 + \left(1 - \frac{4\sqrt{2}\,r_0 m}{q\rho}\right)^{-1} d\rho^2 + \rho^2 d\Omega^2 \tag{5.58}$$

shows that the horizon is at $\rho = 4\sqrt{2}\,r_0 m/q$ and the angular part is spherical; while in the former case, viz, $q_e = 0, q_m = q$, the line element

$$d\tilde{s}_m^2 = -\frac{q^2}{2r_0^2}\left(1 - \frac{4\sqrt{2}r_0 m}{q\rho}\right)dt^2 + \frac{2q^2}{\rho^2}\left(1 - \frac{4\sqrt{2}r_0 m}{q\rho}\right)^{-1} d\rho^2 + 2q^2 d\Omega^2 \tag{5.59}$$

consists of a cylindrical angular part of radius $\sqrt{2}q$. As pointed out in [7], the above metric (5.59) is equivalent to the extremal limit of the magnetically charged GHS black hole (of course, in the string frame [40]) in coordinates that retain the event horizon.

(ii) $a \ll 1$: For small a, $n = 1/(1+a^2) \approx 1$ and one may approximately express the above line element (5.50) and the solutions (5.51) for dilaton and axion fields respectively as

$$ds^2 \approx -\left(\frac{r}{2r_0}\right)^2\left[1 - \frac{2m}{a^2 r}\right]dt^2 + \left(\frac{2r_0}{r}\right)^2\left[1 - \frac{2m}{a^2 r}\right]^{-1} dr^2 + 4r_0^2 d\Omega^2 \tag{5.60}$$

$$\phi(r) \approx \frac{1}{a}\ln\left(\frac{q_m^2 q^4 + 16 q_e^2 r_0^4}{4q^4 r_0^2}\right); \qquad \xi(r) \approx \frac{q_m q_e}{a}\left(\frac{q^4 - 16 r_0^4}{q_m^2 q^4 + 16 q_e^2 r_0^4}\right) \tag{5.61}$$

while the electromagnetic field strength components are approximately given by: $F_{\rm tr} \approx q_e/q^2 dt \wedge dr$; $F_{\vartheta\varphi} = q_m \sin\vartheta d\vartheta \wedge d\varphi$. All the above expressions diverge as $a \to 0$. Therefore the zero limit of a is ill-defined for the above solutions.

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(iii) $a \gg 1$: For large a, $n = 1/(1+a^2) \approx 1/a^2$ and retaining terms upto $\mathcal{O}(1/a^2)$, we can approximately write the above line element (5.50) and the solutions (5.51) for dilaton and axion fields respectively as

$$\begin{aligned} ds^2 &= -\left(\frac{r}{2r_0}\right)^{\frac{2}{a^2}} \left[1 - \frac{2a^2 m}{(a^2-1)r}\right] dt^2 + \\ &+ \left(\frac{2r_0}{r}\right)^{\frac{2}{a^2}} \left[1 - \frac{2a^2 m}{(a^2-1)r}\right]^{-1} dr^2 + r^2 \left(\frac{2r_0}{r}\right)^{\frac{2}{a^2}} d\Omega^2 \end{aligned} \tag{5.62}$$

$$\phi(r) \approx \frac{2}{a} \ln\left(\frac{a q_m}{r}\right); \qquad \xi(r) \approx \frac{q_e}{a q_m}. \tag{5.63}$$

The electromagnetic field strength components are given by: $F_{\rm tr} \approx q_e/(a^2 q^2) dt \wedge dr; F_{\vartheta\varphi} = q_m \sin\vartheta d\vartheta \wedge d\varphi$.

In the limit $a \to \infty$, the line element looks like the standard Schwarzschild line element, although the spacetime geometry originally was non-flat asymptotically for finite value of a. This is, however, understandable because of the suppression factor $1/a$ in the solution for ϕ. Presence of such a factor implies that the dilaton-electromagnetic coupling term $e^{-a\phi}F^2$ in the action, is heavily reduced ($\sim 1/a^2$) for very large a. This automatically ensures a negligible role of both the dilaton and the electromagnetic field in shaping the spacetime geometry, although the dilaton diverges logarithmically as $a \to \infty$. The axion-electromagnetic coupling term $a\xi F {}^\star F$, although present, does not produce any change to the metric solution, since the solution for ξ is approximately trivial and suppressed by the factor $1/a$. In the limit $a \to \infty$, the action in fact reduces to the pure Einstein action supplemented by a constant $\times F {}^\star F$ term. In this case, the solution of the Einstein's equations is uniquely given by the Schwarzschild line element as the term constant $\times F {}^\star F$ is a four divergence term in the action and has no significance. Thus, on the whole, this is a decent check that the Schwarzschild solution indeed serves as a limit to the dilaton-axion black hole solution (5.50) given above, even in view of the fact that such a solution is presumably non-flat asymptotically.

Case $|b| \neq |a|$. As have been the situation for AF spacetimes, here also it is not always possible to obtain analytic closed form black hole solutions when the parameter b differs in magnitude arbitrarily from the parameter a. We consider a particular case corresponding to typically chosen values of a and b (of course $|b| \neq |a|$), for which the values of the undetermined constants n and K_2 can be ascertained in terms of the physical constants q_e, q_m and m in order to satisfy the field equations (5.17). The typical choices are $b \ll 1$ and $a = 1$. The arguments in support of this have already been elucidated in the context of AF black holes in string theory. Similar to case 2, section 5.2, here also we find solutions of the set of equations (5.49) only when the axion ξ is trivial upto order b, i.e., $\xi = \xi_0 + \mathcal{O}(b)$. Neglecting the $\mathcal{O}(b)$ terms, we write down the solutions which can be shown to be given uniquely for $n = 1/2$ and $K_2 = 2q_m^2 q_e^2/r_0^2$:

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$$ds^2 = -\frac{(r-r_+)(r-r_-)}{2r_0 r}dt^2 + \frac{2r_0 r}{(r-r_+)(r-r_-)}dr^2 + 2r_0 r d\Omega^2 \tag{5.64}$$

$$\phi(r) = -\ln\left(\frac{q_e^2}{r_0 r}\right); \qquad \xi(r) = \xi_0 \tag{5.65}$$

$$F_{\mu\nu} \ : \ F_{\rm tr} = \frac{1}{2q_e}dt \wedge dr\,; \qquad F_{\vartheta\varphi} = q_m \sin\vartheta d\vartheta \wedge d\varphi \tag{5.66}$$

where

$$r_\pm = 2\left(m \pm \sqrt{m^2 - \frac{q_e^2 q_m^2}{4r_0^2}}\right). \tag{5.67}$$

There are two horizons located at $r = r_\pm$, where r_+ and r_- are as given by eq. (5.67). The horizons are regular, i.e., all the curvature invariants (i.e., $\mathcal{R}, \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}$ and $\mathcal{R}_{\mu\nu\alpha\beta}\mathcal{R}^{\mu\nu\alpha\beta}$) are well behaved in their locations; $r = 0$ continues to give a physical singularity. The causal structure is similar to the Reissner-Nordström black hole. Under the interchange $q_e \to q_m$, the metric is invariant but the solution for ϕ changes, as does the component $F_{\rm tr}$ of the electromagnetic field. The essential feature of this class of black hole solutions is that the metric explicitly involves the electromagnetic charges q_e and q_m, which is strikingly different from the asymptotically non-flat black hole solution (in Einstein frame) encountered in the case $|b| = |a|$.

6. Conclusion

It is well known that static spherically symmetric spacetimes incorporating black holes do not exist in gravity theories minimally coupled to one or more scalar fields. However, the situation can be remedied either by non-minimal coupling, or the addition of gauge fields, or both. In this paper, we have considered gravity coupled with dilaton (ϕ) and Kalb-Ramond axion (ξ) scalar fields and a Maxwell field, arbitrarily coupled to the scalars. The resultant equations of motion admit of black hole solutions which can be asymptotically flat or asymptotically non-flat. Generically, the black holes are both electrically and magnetically charged. To admit of the latter, the $F_{\mu\nu}{}^\star F^{\mu\nu}$ term has been included in the action as well. In section 2, we considered the most general spherically symmetric metric ansatz and examined the general properties of the equations of motion for the gravitational, Maxwell and scalar fields. We found that, in general, the spacetimes that emerge have both electric and magnetic charges, that ϕ and ξ could be combined into a single 'effective' scalar field ψ, whose solution contains the constants r_0, k and n. Among them, the constant k determines the asymptotic structure of the solution found. In particular, $k = 1$ corresponds to asymptotically flat and $k = 0$ corresponds to asymptotically non-flat solutions. For non-trivial ψ, one also required the condition: $0 < n < 1$.

In section 3, we found the quasi-local mass of the solutions using well known prescriptions. For both AF and AN solutions, a reference spacetime had to be 'subtracted out' in an appropriate sense. The resultant quasi-local mass turned out to be positive and finite.

In section 4, we found the location of the event horizon(s) of the black holes as well as the scalar field ψ in terms of the constants mentioned previously, as well as the electric and magnetic charges. We also examined the generic large r behaviour of Ricci, squared-Ricci and Kretschmann scalars and the possible bounds on the mass for the existence of horizons.

Finally in section 5, we considered a special case of the generic action, inspired by low energy string theory. The latter fixes the arbitrary coupling of the scalar fields to $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}{}^\star F^{\mu\nu}$ terms in the action, upto two dimensionless constants a and b. Such actions have been studied in the context of string theory before. However, in addition to recovering black hole solutions found earlier, we get new solutions to the equations of motion. Once again, they can be asymptotically flat or non-flat. The AF solutions are characterized by the following parameter ranges: for $a = b$, we obtain a new black hole solution, of which for the special case $a = b = 1$ we recover the GMGHS solution, while for $a, b \ll 1$, the Reissner-Nordström solution is recovered. For $b = 1$ and $a \gg 1$, one essentially arrives at the Schwarzschild solution, albeit with non-trivial dilaton and axion fields. For $a = 1$ and $b \ll 1$ on the other hand, we obtain a black hole with two horizons, which is both electrically and magnetically charged, and of which the Reissner-Nordström and GMGHS black holes are special cases. Similarly, for the AN scenario, $a = b$ corresponded to a new black hole solution with both electric and magnetic charges. For either electric or magnetic charge being zero, we recover the AN dilaton black hole solution found in [7]. Finally, for $a = 1$ and $b \ll 1$, we obtain a dyonic black hole with two horizons.

Thus in this work we present new black hole solutions for Einstein-Maxwell-scalar field systems inspired by low energy string theory. It would be interesting to study its generalizations to rotating black holes. It would also be interesting to study the phenomenological consequences of these black holes, such as gravitational lensing as well as the behaviour of particle geodesics, and whether there is any hope of testing these in astrophysical observations and/or observations in future high energy colliders (if the higher dimensional Planck energy is indeed small). Similarly, thermodynamic and stability properties (including quasi-normal modes) could be of interest. We hope to report on this elsewhere.

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